

Mathematics for Precision Shooters

Recently my old friend Steve Langford of Millett Sights and I discussed the many formulas, measurements, math shortcuts and rules-of-thumb that are of value to precision shooters. He and I jotted down quite a collection which I'm passing on to you:

Equivalents and Conversion Factors

These are worth knowing so you can convert units of measurement -- especially when using a meters-based map -- or employing a scope that uses metric adjustments.

- One centimeter equals 10 millimeters
- One centimeter equals 0.3937 inches
- To convert centimeters to inches, multiply the inches by 2.54
- One inch equals 2.54 centimeters
- To convert inches to centimeters, multiply the inches by 0.3937
- 1 Yard equals 0.9144 Meter
- 100 Yards equals 91.44 Meters
- To convert yards to meters, multiply the yards by 0.9144
- 1 Meter equals 1.0936 Yards or 39.37 Inches

- 100 Meters equals 109.36 Yards

- To convert meters to yards, multiply the meters by 1.0936

Minutes-of-Angle and Mil-Dots

The Minutes-of-Angle is the standard unit-of-measurement for elevation and windage on most riflescopes in America, while the mil (or milradian) is the unit-of-measurement found in mil-dot reticles. In a previous column I dealt with both mils and MOAs to a great extent, explaining how to calculate measurements and use them in various ways. So here we're only contrasting mils and MOAs at 100-yard increments all the way to 1000 yards to give you a sense of how these compare.

	One Minute of Angle Equals	One Mil Equals
100 Yards	1.0 Inch	3.6 Inches
200 Yards	2.0 Inches	7.2 Inches
300 Yards	3.0 Inches	10.8 Inches
400 Yards	4.0 Inches	14.4 Inches
500 Yards	5.0 Inches	18.0 Inches
600 Yards	6.0 Inches	21.6 Inches
700 Yards	7.0 Inches	25.2 Inches
800 Yards	8.0 Inches	28.8 Inches
900 Yards	9.0 Inches	32.4 Inches
1000 Yards	10.0 Inches*	36.0 Inches

**By the time you reach 1000 yards, an actual Minute of Angle equals 10.47 inches*

Mil Ranging Estimates

Here are a variety of ways to calculate distances using a mil-dot reticle. The first technique is especially useful because it enables you to base your calculation on an object that is measured in inches rather than fractions of a yard.

TO FIND A DISTANCE IN YARDS: An object of known size IN INCHES x 27.8 divided by the object's size IN MILS equals its distance IN YARDS. Example: you know an object at the target's distance is 12 inches wide; you measure the object's width in your scope's mil scale and find it is 2 mils wide -- therefore, 12×27.8 divided by 2 equals 166.8 meaning the target is 166.8 yards away. [*That "27.8" is simply a constant* – don't get wrapped up about why it's 27.8 but DO NOT use this constant for computing a distance using meters! It won't work!]

Here's another way **TO FIND A DISTANCE IN YARDS:** At the target, identify an object of known size IN YARDS and then measure it in mils. Now, multiply its estimated size IN YARDS by 1000 then divide that by the object's measured size in mils. Example: You see a fencepost that you know is two yards high; measuring it in your mil-dot reticle you see it is 4 mils high. Thus, multiple its 2-yard height by 1000, which equals 2000. And since that post measured 4 mils in your reticle, you divide the 2000 by 4, so you know it is 500 yards away.

And to find a distance **IN METERS:** Place your mil-dot reticle on an object whose size is known size in inches. Measure it in mils. Now, Take the object's size IN

INCHES and divide by the constant 39.4 and then multiply the result by 1000 and then divide that by the object size IN MILS. Try this example: You see an old tire near your target, and know the tire is 34 inches wide. Measured in your mil reticle, the tire is 1.5 mils wide. So, you divide the 34 inches by the constant of 39.4 to yield 0.86. Multiply that by 1000 to get 860, and divide that by 1.5. Therefore, your target is 573 meters away.

Calculating Elevation and Windage Changes

Retired Navy SEAL Master Chief Jim Kauber, with whom I instructed at Gunsite Training Center, developed this formula for determining sight changes at various distances.

Let's say you've fired a shot at 436 yards and you observe it strike 15 inches low. How much do you adjust your elevation? Here's Jim's formula for determining a scope correction in Minutes of Angle:

$$\frac{\text{Correction (in inches)}}{\text{Distance (in hundreds of Yards)}} = \text{MOA Change}$$

Now let's use the numbers from our example:

$$\begin{array}{l} 15 \text{ (inches)} \\ \text{-----} \\ \text{4.36 (hundreds of Yards)} \end{array} = 3.44 \text{ MOA (rounded to 3-1/2 MOA)}$$

Thus on a scope with 1/4-minute increments, we'd achieve this 3-1/2 MOA change by raising our elevation fourteen clicks.

Here are ways to find the third factor when any two factors are present:

$$\begin{array}{l} \text{Inches} \\ \text{-----} \\ \text{Distance} \end{array} = \text{MOAs}$$

OR

$$\begin{array}{l} \text{Inches} \\ \text{-----} \\ \text{MOAs} \end{array} = \text{Distance}$$

OR

$$\text{MOAs} \times \text{Distance} = \text{Inches}$$

To keep all these variations straight, Gunsite precision rifle graduate Frank Zaluga devised his own formula graphic, which he forwarded to me. Frank dubbed it, "The Doctor is in."

IN (*inches*)

M **D**

IN = Inches of change on target

M = Minutes of Angle

D = Distance in hundreds of yards

To use Frank's method, just place your fingertip over the factor you want to determine, which tells you to multiply or divide the other two factors, to learn the third factor. Let's say you want to see how many inches 5 MOAs are at 200 yards: Just place a fingertip over "IN" and you see that you must *multiply* the MOAs by the Distance, which means 5 MOAs times 2 (*Distance in hundreds of yards*). Therefore, the answer is 10 inches. And if you're trying to determine how many MOAs to crank your scope when you want to raise the bullet impact ten inches at 200 yards: put a fingertip on the "M" (for MOA), and you see you must divide the inches by the distance. Frank's graphic formula is simple and easy to remember.

An Old Marine Corps Method

Here's an old USMC formula for estimating windage compensation, in Minutes of Angle. This only applies to .30-caliber rounds, such as the .308 Winchester and .30-06 Springfield. In use since the days of the excellent 1903 Springfield rifle,

this formula is only employed for targets 500 yards or closer, after which the math constant changes:

$$\frac{\text{Range (hundreds of yards)} \times \text{ Windspeed (MPH)}}{15 \text{ (a math constant)}} = \text{MOA Windage}$$

For example, your target is 300 yards away, and there's a 10 MPH crosswind.

$$\frac{3 \times 10 = 30}{15} = 2 \text{ Minutes of Angle}$$

If it's a full-value (90 degree) crosswind, click-in the two Minutes of Angle and aim dead-on; if the wind is oblique, proportionally reduce your clicks. This formula works well except when a target is farther than 500 yards – after that, the Math Constant increases, as shown below:

At 600 Yards, Divide by 14

At 700 and 800 Yards, Divide by 13

At 900 Yards, Divide by 12

At 1000 Yards, Divide by 11

Compensating for Headwinds and Tailwinds

In theory, a headwind slightly increases drag on your bullet and thereby reduces its velocity, while a tailwind has the opposite effect. Thus, a headwind requires that you slightly raise your scope reticle, and a tailwind dictates that you slightly lower it.

This formula, cited by W.W. Greener, is:

$$\frac{\text{Wind Velocity (MPH)} \times \text{Distance (Hundreds of Yards)}}{4 \text{ (Math Constant)}} = \text{Yards (Range Change)}$$

Let's see how exact this small compensation can be by calculating the adjustment for a 10 MPH tailwind, when our target is at 300 yards

$$\frac{10 \times 3}{4} = 7.5 \text{ Yards}$$

Since it's a tailwind, we subtract these 7.5 yards from the 300, and set our sight as if the target is 292.5 yards away - which translates (for a .308 Winchester firing 168-gr. BTHP) to *just one click -- one tiny 1/4-Minute of Angle click!* Had it been a headwind, we would have added that 7.5 yards to the actual distance, and fired as if the target were 307.5 yards away.

This may seem like splitting hairs, but headwind/tailwind compensation increases with distance because the bullet slows and its trajectory becomes a plunging arc. At 900 yards, that same, mild, 10 MPH headwind or tailwind, would require a seven click correction to place our .308 bullet on center-target. And recall that at such extreme range, the bullet is plunging so sharply that even a small elevation error can cause a complete miss.

Adjusting for Temperature Changes

This is another elevation adjustment, dictated by how much temperature variation can change the trajectory of your bullet. Because powder inside your cartridge burns at a higher rate when it's warm, and slower when it's cold, your rounds will strike low in cold weather and high in hot temperatures. The following best applies to .308 and .30 caliber rounds:

When the temperature changes 20 degrees from your zero temp, apply 1 MOA at 300 yards;

When the temperature changes 15 degrees from your zero temp, apply 1 MOA at 600 Yards;

When the temperature changes 10 degrees from your zero temp, apply 1 MOA at 1000 Yards

U.S. Army tests measured the following effects of changing temperatures on .308 ammunition,

Degrees Fahrenheit	Muzzle Velocity	Bullet Drop at 600 Yards (200-Yard Zero)
-10	2400 fps	-109 inches
+25	2500 fps	-100 inches
+59	2600 fps	-91 inches
+100	2700 fps	-84 inches

A century ago, British firearms authority W.W. Greener developed a formula based upon a "standard" temperature of 60 degrees Fahrenheit, which is very close to today's 59-degree F. ballistic table standard:

$$\frac{\text{Degrees } +/- \text{ 60 Degree Standard}}{\text{10 (Math Constant)}} \times \text{Distance (Hundreds of Yards)} = \text{Range Change for Temperature}$$

When the temperature is less than 60 degrees, add the result to your actual target distance; when it's above 60 degrees, subtract this distance from the actual distance. Here's an example: The temperature is 90 degrees F. (30 degrees hotter than Greener's 60-degree F. standard), and your target is 500 yards away.

$$\frac{30}{10} \times 5 = 15$$

Deduct this distance (15 yards) from the 500 yards, then set your sights as if the target is 485 yards away, aim dead-on, and fire. Altitude Effects

Altitude Effects

Since air is thinner at higher altitudes, there's less drag on a bullet and therefore, in relative terms, a bullet flies faster with a flatter trajectory. Standard ballistic tables assume a sea-level altitude, so already there may be a variance from these tables and the altitude at which you zero your rifle.

Concern about altitude change usually involves travel - say you regularly fire your rifle near St. Louis at an altitude of 450 feet, then you go to Colorado to hunt elk at 10,000 feet. What's the difference? The basic rule-of-thumb is, add or subtract one MOA for every 5000 feet of elevation change.

But what about changes in barometric pressure which can vary day-by-day, even hourly, when high- or low-pressure fronts pass through? I don't think there's enough variance to change your elevation except in extreme weather changes; yet, W.W. Greener, in his 1901 classic, *Sharpshooting for Sport and War*, suggested, "One inch of barometric pressure is equal to 15 degrees Fahrenheit change in temperature. Treat a rise in barometer as a rise of the thermometer in the above proportion."

Trajectory Changes for Barrel Length

When you consult an ammo manufacturer's rifle ballistic tables, you're actually looking at the velocity when fired through a 24-inch barrel. The degree to which your barrel is shorter or longer will respectively decrease or increase muzzle velocity, and thus require elevation changes, a significant factor since some rifles have 26-inch and even 20-inch barrels.

Here are the three rules of thumb, as disseminated by Remington Arms

Company:

- - For cartridges generating a muzzle velocity of 2000 - 2500 fps, each inch a barrel is longer or shorter than 24 inches changes velocity 10 fps;
- - For cartridges generating 2500 - 3000 fps, one inch will vary velocity 20 fps;
- - High velocity cartridges, generating 3000 - 3500 fps, will change muzzle velocity 30 fps per inch of barrel.

While these velocity differences seem minimal - at 100 yards there's almost no noticeable change in trajectory - the effect becomes significant at long range. For example, the .308, 168-gr. BTHP Match round will impact six inches lower at 500 yards when fired from a 20-inch barrel, then when fired from a 24-inch barrel.

After determining your round's muzzle velocity for your barrel length, consult a good ballistic software program to chart how it changes the bullet's trajectory.

A Quick-Fix for Up/Down Situations

When your target is uphill or downhill, your round will impact high if you aim

dead-on. (The explanation is complex and worthy of a future column, but for now, just trust me.) There are several methods to determine the amount of compensation, but the quickest involves these simple Rules of Thumb:

- - For a target 45 degrees up or down, multiply the actual distance by 0.7, set your scope elevation for this distance and aim dead-on;
- - When the target's 30 degrees up or down, multiply its range by 0.9, set your scope for this distance and aim dead-on;
- - If the target angle is less than 30 degrees, aim dead-on.

Another Rule-of-Thumb, suggested to me by Sergeant Neal Terry, a longtime sniper instructor and police SWAT sniper, is to draw an invisible vertical line through the up or down target, and where this line intersects the earth's surface, range it! Disregard the angle, ignore the direct distance from muzzle-to-target, concentrate only on adjusting elevation for the distance to where that vertical line touches the earth. Compensate for that distance, aim dead-on and fire. We've worn out several pads of paper comparing Neal's math to other kinds of up/down calculations - it works and it's the essence of simplicity!

Exit Pupil and Lowlight Observation

At dusk and at night, your eye's pupil dilates, or widens, to 6 or 7 millimeters, so you can see better. Ideally, your riflescope or binoculars should generate a cone of light of this measurement to allow the best lowlight viewing. How to calculate? Divide the objective (forward) lens diameter in millimeters by the optical device's magnification. Say you have fixed-power 7x binoculars with 35 mm objective lenses -- thus 35 mm divided by 7 equals an exit pupil of 5mm, meaning the cone

of light reaching your eyes will be 5 millimeters wide. Not quite as wide as your eye pupils. Thus, 7 x 50 mm binoculars would be better, because the exit pupil would be 7mm. Here are the best riflescope magnifications for shooting in low light, depending upon the size of your objective lens.

Objective Lens Diameter	Best Night Magnification
56 mm	8-9x
50 mm	7-8 x
44 mm	6-7 x
40 mm	5-6 x

Calculating a Barrel's Rate-of-Twist

Known as the Greenhill Formula, here's how to calculate the most compatible rifling rate-of-twist for a given projectile. First, determine the LENGTH of your bullet in "calibers" which is a measurement in hundredths-of-an-inch, the same way we measure bullet diameters. (ie., a .50 caliber bullet is 0.50 inches in diameter). Now divide the length in calibers/hundredths by its diameter caliber. Next, take this figure and divide by 150, which is simply a math constant. And finally, take that result and divide again by the diameter caliber and you'll have the rate-of-twist in inches. Here's an example: Your .30 caliber, 200-grain bullet measures 1.35 calibers (or inches) in length. Take the 1.35 length and divide by the .30 diameter, which equals 4.5. Divide 150 (the constant) by that 4.5 which equals 33.33 Now take that and multiply by the caliber diameter, thus, 33.33 multiplied by .30 caliber equals 9.999 or a 1-in-ten inch rate of rifling twist.

Bullet Length Versus Diameter

Today's Very Low Drag bullets are disproportionately longer than Whitworth recommended, a reflection, I think, of our higher velocities leading us ever farther from round balls, and further from rules-of-thumb of past ages.